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TROPHIC CASCADES SCENARIO OF A DETRITUS-BASED STAGE-STRUCTURED MODEL IN MARINE ECOLOGY

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Abstract.

The impact of multiple stresses due to human activities on the biosphere makes the environmental science a challenging field of study. This is a vital step towards sustainable management in ecology. It is particularly important to consider the coastal salt marsh estuarine system because it is one amongst the most biologically productive areas in the world and is used by human beings for a variety of purposes. The areas under discussion are now experiencing changes in nutrient loading, species composition and sea level raise which are 5-10 fold higher in comparison to that which was measured over the last century. This is undoubtedly affecting the productivity and sustainability of coastal regions. Here, the mathematical model of a detritus-based ecosystem with stage-structure and selective harvesting which is mainly found in Sunderban Mangrove area in India is considered. At the positive stationary state, the local and global stability due to discrete time delay and stochastic perturbation is analyzed. The exploitation of the prey is controlled by a regulatory agency by imposing a tax per unit biomass of the detritus and the optimal harvesting policy is achieved by using Pontryagin's maximum principle. It is very interesting that the time delay has a great role in the real ecology by inducing a stable equilibrium into an unstable one. Finally numerical simulations are carried out to compare the analytical results.

Keywords: Detritus; detritivorus; stage structure; stability; time delay; stochastic perturbation; harvesting.

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1 Introduction

Despite the urgency of the problems in a real ecosystem, the ecology cannot predict the response of the salt marshes to nutrient enrichment and biotic impoverishment. The theory which describes the relative importance of nutrients (bottom-up) or species composition (top-down) on ecosystem function has never been tested in detritus-based aquatic systems earlier. The salt marshes exemplify detritus-based aquatic ecosystems in which the food-web base is on the consumption of non-living organic matter (detritus). In addition, detritus is important in creating the physical structure of the system by the formation of peat. The combined effects of nutrient enrichment and biotic impoverishment will have its impact on ecosystem function and sustainability, because of nonlinear feed-backs among components of the ecosystem. The term mangrove refers to an ecological system which dominates the world's tropical and subtropical coasts. It is a fact that energy and nutrients are stored in leaves of mangroves. Mangroves are of interest to biologists and scientists, because of their diversity and productivity; and furthermore, they are great source of goodness prove to be beneficial to humans. In addition, they are exploited for timber that is widely used for construction and as firewood. Their use ranges from the casual collection of fallen wood to the large-scale industrial charcoal production. The latter form usually coincides with the use of intensive mangrove plantations, such as the ones in Sundarban, India. The foliage, at times is grazed or harvested for feeding domestic animals.

The perspective of energy flow in ecological systems, comprise of four general types of heterotrophs. Herbivores are called primary consumers because they eat only plants. Carnivores are called secondary consumers because the eat herbivores. There is also a group of tertiary consumers called omnivores because they eat both herbivores and carnivores. The other type of consumer that is critical to energy flow in the ecosystems comprise of organisms that feed on dead plants and animals. These are called detritivores which has the main role in this paper.

Some of the most significant changes which are visible involve in global warming, sea level rise, widespread nutrient enrichment and evolution of species. Anthropogenic nitrogen fixation has recently exceeded the natural biotic fixation and will continue to increase for a predictable future [49]. The dynamics of species through overfishing, extinction and exotic species introduction have large, but partially understood effects on the ecosystems of forests, lakes, streams, and estuaries [3, 53, 18, 22, 17]. Understanding and predicting how multiple stresses affect the sustainability of ecosystems is one of the most crucial challenges in environmental biology and the first step towards management. Mangroves are critical, not only for sustaining biodiversity, but also for their direct and indirect benefits to human activities. Mangroves have long functioned as a storehouse of materials providing food, medicines, shelter and tools. Fish, crabs, shellfish, prawns and edible snakes and worms are found there. The fruit of certain species including the nypa palm can be eaten after preparation along with the nectar of some of the flowers. The best honey is considered to be that produced from mangroves, particularly the river mangrove Aegiceras corniculatum.

Modelling is an essential tool used throughout to test our predictions and to scale-up the consequences of changes in nutrients and trophic structure to encompass larger spatial and longer temporal-scales. The modelling objectives are essential to develop a fine scale model that incorporates a dynamic ecological processes for creek watersheds and use this as a basis to predict landscape responses to a varying nutrient and trophic structure regimes. An important challenge is to determine the amount of detail, including the resolution of physical space and biotic interactions, necessary to model landscape responses [14, 45, 28]. Although it seems intuitive that more a model incorporates, the processes and mechanisms known to occur in the real world situation; the close it should come to predict the ecosystems impact in particular perturbation. Often, more complex, data-intensive models are less stable and more difficult to test [37]. An additional difficulty is that many management questions require for landscape answers, but most of the measurements of the biotic process are by necessity at smaller scales. Energy and nutrients are assimilated and stored in the leaves of mangrove trees. Being a detritus-based ecosystem, leaf litter from these trees provides a base for adjacent aquatic and terrestrial food webs. Because most of the energy and nutrients are stored in biomass rather than being free in the water or substrate where the species diversity of these swamps is directly dependent on the primary productivity by mangrove plants.

The productivity of mangrove ecosystems also supports fisheries through the export of carbon. Few fish species are permanent residents in mangroves, but numerous marine species uses the mangroves as nursery grounds. Mangrove swamps also provide feeding grounds for juvenile and sub adult reef fishes. As a result, mangrove-assimilated energy and nutrients are exported to surrounding coral reefs. It is known that an estimated 75 percent of the fish which is caught commercially spend some time in the mangroves or it is dependent on a food chain which can be traced back to these coastal forests. Fisheries are not the only benefactors of mangrove services for the shrimp trawling industry is also deeply dependent on the nursery function of the mangroves.

The sustainability of coastal ecosystems in the face of widespread environmental change is current issue of pressing concern throughout the world [17]. Coastal ecosystems are a dynamic interface between terrestrial and oceanic systems and happen to be the most productive ecosystems in real world. Coastal systems probably serve more human beings compared to other ecosystem [17]. They have always been valued for their rich bounty of fish and shellfish. Coastal areas are also the sites of the nation's and the world's most intense commercial activity and population growth worldwide which may approximately amount to 75 percent of human population who live in coastal regions [50]. The coastal environment is continually changing because of the natural variability of environmental drivers because of their sensitivity to small changes in the sea level which is a part of the question in the nonlinear phenomena. In the recent times, changes are observed in the environment have been accelerated towards nutrient enrichment, species composition and sea level due to man-made alterations at a place that is outside the bounds of natural variability.

Nutrient enrichment is a challenging current premier issue for coastal researchers and managers for the last three decades [8]. Our goal is to understand the eutrophication which has been developed through observation of estuaries that are undergoing cultural nutrient enrichment, particularly nitrogen [8, 30]. The consequences of enhanced nitrogen delivery in estuaries follow the classic response of ecosystems to stress (e.g., altered primary producers, altered nutrient cycles and loss of secondary producer species and production) [15, 21, 42, 35]. Although most researchers treat the nutrient enrichment as a stand-alone stress; recent studies suggest that the responses at estuaries to enrichment may depend on species composition [8, 10, 32, 46]. At the same time, biotic impoverishment is reducing the abundance of species or the loss of species due to man-made distraction [32, 44, 9]. Most of the changes in species composition affect estuaries directly through food web interactions such as predation or competition [15] or indirectly by altering the rates or pathways of nutrient cycle [12, 48].

Unfortunately, existing ecological theory is unable to predict the responses of coastal salt marsh ecosystem to the combined effect of nutrient enrichment and biotic impoverishment as it is a highly nonlinear phenomena. The theory describes the relative importance of nutrients (bottom-up) or species composition (top-down) on ecosystem function which has never been tested in detritus-based aquatic systems, which raises a challenging issue in this paper. The bottom-up idea originated from the observation that nutrient availability sets a general quantity of primary productivity, whereas other studies have already shown that species composition, particularly of top consumers, has a marked cascading effect on ecosystems [36, 31, 13, 38]. Most examples of trophic cascades are in closed aquatic ecosystems with fairly simple direct algal grazing food webs [47, 51]. The rarity of trophic cascades in terrestrial systems has been attributed to the importance of detrital food webs, where omnivory is common [39, 40]. In detritus-based systems [52] where the food web is basically based on the consumption of dead organic material (detritus) by omnivorous species and detritus-based aquatic ecosystems, like salt marshes, bogs, swamps, have been considered bottom-up or physically controlled ecosystems. This assumption, however, has not been tested earlier. Our study give a documentation of the trophic cascades in theoretically unlikely systems such as tropical forests and open ocean [38] indicating the need for direct testing of controls in different ecosystems.

Harvesting plays a major role on the dynamic evolution of a population and also depends on the nature of the applied harvesting strategy where in the long run stationary density of the population is significantly smaller than the long run of a stationary density of a population in the absence of harvesting [4]. In this detritus-based model of a mangrove ecological system, the level of harvesting effort expands or contracts according as the net economic revenue to the owner of the system that is positive or negative. Any model which includes this interaction between the net economic revenue and the harvesting effort is called the dynamic reaction model. Recently, seasonal harvesting, taxation, lease of property rights etc. are usually considered as governing instruments for the regulation of exploitation of biological resources which have become a major problem. Out of all such regulating options, taxation plays a major role and is superior because of its economic flexibility. It worthwhile to mention that there are other researchers [5, 29, 6, 41, 24, 16] who studied elaborately about the harvesting problems with taxation as a control instrument.

The recent developments in bio-economics such as the prey-predator models play a major role in the nonlinear world. Researchers [7, 25, 11, 33] have earlier discussed the prey-predator system with harvesting, but they have not considered the stage-structure of the species with delays and stochastic perturbation. Some of the stage-structure models are considered [26, 27, 43] without delays and stochastic version. It is also difficult to analyze the stability of this type of mathematical model in population biology by incorporating time delay and stochasticity. Researchers [54, 19, 55] have studied about the dynamics of stage-structured prey-predator model with discrete delays only. The prey-predator model with stage-structure for prey is analyzed [55] and it obtains the necessary condition for the system stability. Authors [19] have discussed about a stage-structured prey-predator model with a systematic computational study. In the present paper, we have studied the detritus-based stage-structured stochastic delayed model in mangrove ecosystem where the harvesting of prey species is considered. We categorized this paper under formulation of the model, equilibrium analysis, stability analysis with and without time delay, stability with stochastic perturbation, optimal harvesting, numerical simulation and concluding remarks.

2 The mathematical model

We consider the detritus based model ecosystem. The model equations for this ecological system are constructed by the following system of non linear differential equations.

$$\frac{\mathrm{d}x}{\mathrm{d}t} = K - \alpha x - \beta xy + d_3 z - qEx, \tag{2.1}$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y(-d_1 + c_1 x - c_2 z), \tag{2.2}$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = -d_2 z + c_3 y(t-\tau) z(t-\tau), \tag{2.3}$$

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \lambda\left[(p-\sigma)qx - \gamma\right]E, \tag{2.4}$$

where $x(t)$ represents density of biomass of the plant litter of the mangroves plants after decomposition (detritus) at time t, $y(t)$ represents biomass of micro organisms (detritivorus) at time t, $z(t)$ represents biomass density of predators of detritivorus at time t, $E = E(t)$ represents harvesting effort at any time t , K is constant input of the detritus, q is the catch ability co-efficient, α is washed out rate of the detritus, β is the conversion factor, c_1 is the conversion rate of detritus to detritivores, c_2 is the maximum uptake rate of detritivores, c_3 is the specific growth rate of predators detritivores, d_1 is the death rate of micro-organisms (detritivores), d_2 is the death rate predators of detritivores, d_3 is the detritus recycle rate after the death of predators of detritivorus, λ is the stiffness parameter used to measure the harvest effort, p is the fixed price per unit of prey species, γ is the fixed cost of harvesting per unit of effort, σ is the tax per unit biomass of the prey.

3 Equilibrium analysis

The steady states of the model (2.1)–(2.4) are given by: (i) $G_0(0, 0, 0, 0)$, (ii) $G_1(\bar{x}, \bar{y}, 0, 0)$, (iii) $G_2(x^{\phi}, y^{\phi}, z^{\phi}, 0)$, (iv) $G_3(x^*, y^*, z^*, E^*)$.
Case (i): $G_0(0, 0, 0, 0)$: This steady state always exists.
Case (ii): $G_1(\bar{x}, \bar{y}, 0, 0)$: If $\bar{x}$ and $\bar{y}$ are the positive solutions of $\frac{\mathrm{d}x}{\mathrm{d}t} = 0$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = 0$, then we get

$$\bar{x} = \frac{d_1}{c_1}\,;\, \bar{y} = \frac{c_1 K}{\beta d_1} - \frac{\alpha}{\beta}. \tag{3.1}$$

For $\bar{y}$ to be positive, we must have

$$\frac{c_1 K}{\beta d_1} > \frac{\alpha}{\beta}. \tag{3.2}$$

Case (iii): $G_2(x^{\phi}, y^{\phi}, z^{\phi}, 0)$: If x^{ϕ} , y^{ϕ} and z^{ϕ} are the positive solutions of $\frac{\mathrm{d}x}{\mathrm{d}t} = 0\,;\, \frac{\mathrm{d}y}{\mathrm{d}t} = 0\,;\, \frac{\mathrm{d}z}{\mathrm{d}t} = 0$, then

$$y^{\phi} = \frac{d_2}{c_3};\, z^{\phi} = \frac{Kc_1c_3 - \alpha c_3 d_1 - \beta d_1 d_2}{\alpha c_2 c_3 + \beta d_2 c_2 - d_3 c_1 c_3}\,;\, x^{\phi} = \frac{c_3\left(Kc_2 - d_1 d_3\right)}{\alpha c_2 c_3 + \beta d_2 c_2 - d_3 c_1 c_3}.$$

For z^{ϕ}, x^{ϕ} to be positive, we must have

$$K\,c_2 > d_1\,d_3 \;\;;\;\; c_2(\alpha\,c_3 + \beta\,d_2) > d_3 c_1 c_3. \tag{3.3}$$

Case (iv): $G_3(x^*, y^*, z^*, E^*)$: If x^*, y^*, z^*, E^* are the positive solutions of $\frac{\mathrm{d}x}{\mathrm{d}t} = 0$; $\frac{\mathrm{d}y}{\mathrm{d}t} = 0$; $\frac{\mathrm{d}z}{\mathrm{d}t} = 0$; $\frac{\mathrm{d}E}{\mathrm{d}t} = 0$, then

$$x^* = \frac{\gamma}{(p-\sigma)q}\,; y^* = \frac{d_2}{c_3}; z^* = \frac{1}{c_2}\left[\frac{c_1\gamma}{(p-\sigma)q} - d_1\right];$$

$$E^* = \frac{1}{qx^*}\left[K - x^*\left(\alpha + \frac{\beta d_2}{c_3}\right) + d_3 z^*\right].$$

For z^*, E^* are to be positive, we must have

$$c_1\gamma > d_1 \;\; ; \;\; p > \sigma \; ; \;\; K + d_3\, z \; > \; x\,(\alpha + \beta\, y). \tag{3.4}$$

From the expression of x^*, y^*, z^* and E^*, we can easily verify that the equilibrium point (x^*, y^*, z^*, E^*) exists provided that the tax σ lie in the range $\sigma_{\min} \le \sigma \le \sigma_{\max}$, where $\sigma_{\min} = p - \frac{c_1\gamma}{d_1 q}$; $\sigma_{\max} = p - \frac{\gamma(c_1c_3d_3 - \alpha c_2c_3 - \beta c_2 d_2)}{(d_1d_3 - kc_2)qc_3}$ and the parameters must satisfy the following condition: $p > \frac{c_1\gamma}{d_1 q} > \frac{\gamma(c_1c_3d_3 - \alpha c_2c_3 - \beta c_2 d_2)}{(d_1d_3 - kc_2)qc_3}$.

4 Stability analysis without time delay

In the absence of discrete time delay, we now investigate the stability of the model around the interior equilibrium. In the absence of delay, the model (2.1)–(2.4) is reduced to the following form:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = K - \alpha x - \beta xy + d_3 z - qEx, \tag{4.1}$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y(-d_1 + c_1 x - c_2 z), \tag{4.2}$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = -d_2 z + c_3 yz, \tag{4.3}$$

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \lambda\,[(p-\sigma)qx - \gamma]\,E. \tag{4.4}$$

The Jacobian matrix of the model (4.1)–(4.4) is

$$\begin{pmatrix} -\alpha - \beta y - qE & -\beta x & d_3 & -qx \\ c_1 y & -d_1 + c_1 x - c_2 z & -c_2 y & 0 \\ 0 & c_3 z & -d_2 + c_3 y & 0 \\ \lambda E(p-\sigma)q & 0 & 0 & \lambda[(p-\sigma)qx - \gamma] \end{pmatrix}. \tag{4.5}$$

At the interior equilibrium point, we have

$$-\alpha - qE = -\frac{K}{x} + \beta y - \frac{d_3 z}{x}; -d_1 = -c_1 x + c_2 z; -d_2 = -c_3 y\,; (p-\sigma)q = \frac{\gamma}{x}. \tag{4.6}$$

At the interior equilibrium point, (4.5) becomes

$$\begin{pmatrix} -\frac{K}{x} - \frac{d_3 z}{x} & -\beta x & d_3 & -qx \\ c_1 y & 0 & -c_2 y & 0 \\ 0 & c_3 z & 0 & 0 \\ \frac{\lambda\gamma E}{x} & 0 & 0 & 0 \end{pmatrix}. \tag{4.7}$$

The characteristic equation of (4.7) of the system (4.1)–(4.4) is given by

$$\begin{vmatrix} -\frac{K}{x} - \frac{d_3 z}{x} - \mu & -\beta x & d_3 & -qx \\ c_1 y & -\mu & -c_2 y & 0 \\ 0 & c_3 z & -\mu & 0 \\ \frac{\lambda\gamma E}{x} & 0 & 0 & -\mu \end{vmatrix} = 0. \tag{4.8}$$

This implies that

$$\mu^4 + \mu^3 \left(\frac{K}{x} + \frac{d_3 z}{x}\right) + \mu^2 \left(\lambda\gamma E q + c_2 c_3 y z + \beta c_1 x y\right)$$

$$+\mu \left(\frac{K c_2 c_3 y z}{x} + \frac{c_2 c_3 d_3 y z^2}{x} - c_1 c_3 d_3 y z\right) + \lambda\gamma E q c_2 c_3 y z = 0. \tag{4.9}$$

Equation (4.9) is in the form of

$$\mu^4 + A\mu^3 + B\mu^2 + C\mu + D = 0, \tag{4.10}$$

where $A = \frac{K}{x} + \frac{d_3 z}{x}$, $B = \lambda\gamma E q + c_2 c_3 y z + \beta c_1 x y$; $C = \frac{K c_2 c_3 y z}{x} + \frac{c_2 c_3 d_3 y z^2}{x} - c_1 c_3 d_3 y z$; $D = \lambda\gamma E q c_2 c_3 y z$.

By Routh-Hurwitz criteria, the model (4.1)–(4.4) is locally stable around the interior equilibrium point, if the following conditions hold: $A > 0$; $AB - C > 0$; $C(AB - C) > A^2 D$. Here

$$A = \frac{K}{x} + \frac{d_3 z}{x} > 0, \tag{4.11}$$

$$AB - C = \frac{K\,\lambda\,\gamma\,E\,q}{x} + K\,\beta\,c_1 y + \frac{\lambda\,\gamma\,E\,q\,d_3 z}{x} + \beta\,c_1 d_3 y\,z + c_1 c_3 d_3 y\,z > 0, \tag{4.12}$$

$C(AB - C) - A^2 D = \frac{K^2 \beta\, c_1 c_2 c_3 y^2 z}{x} + \frac{K\beta\, c_1 c_2 c_3 y^2 z^2}{x} + \frac{K\beta\, c_1 c_2 c_3 y^2 z^2}{x} + \frac{K\, c_1 c_2 c_3^2\, d_3 y^2 z^2}{x} + \frac{\beta\, c_1 c_2 c_3\, d_3^2 y^2 z^3}{x} + \frac{c_1 c_2 c_3^2\, d_3^2 y^2 z^3}{x} - \frac{K\,\lambda\,\gamma\,E\,q c_1 c_3\, d_3 y z}{x} - K\beta\, c_1^2 c_3 d_3 y^2 z - \frac{\lambda\,\gamma\,E\,q c_1 c_3 d_3^2 y z^2}{x} - \beta\, c_1^2 c_3 d_3^2 y^2 z^2 - c_1^2 c_3^2 d_3^2 y^2 z^2 > 0$

$$\text{if } z > \frac{c_1}{c_2}\,x \text{ and } y > \frac{\lambda\,\gamma\,E\,q d_3}{K\beta\,c_2}. \tag{4.13}$$

To find the condition for global stability at $G_3(x^*, y^*, z^*, E^*)$, we construct the following Lyapunov function:

$$V(x, y, z, E) = \left[(x - x^*) - x^* \ln \frac{x}{x^*}\right] + l_1 \left[(y - y^*) - y^* \ln \frac{y}{y^*}\right]$$

$$+ l_2 \left[(z - z^*) - z^* \ln \frac{z}{z^*}\right] + l_3 \left[(E - E^*) - E^* \ln \frac{E}{E^*}\right]. \tag{4.14}$$

Then

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \left(\frac{x - x^*}{x}\right) \frac{\mathrm{d}x}{\mathrm{d}t} + l_1 \left(\frac{y - y^*}{y}\right) \frac{\mathrm{d}y}{\mathrm{d}t} + l_2 \left(\frac{z - z^*}{z}\right) \frac{\mathrm{d}z}{\mathrm{d}t} + l_3 \left(\frac{E - E^*}{E}\right) \frac{\mathrm{d}E}{\mathrm{d}t}.$$

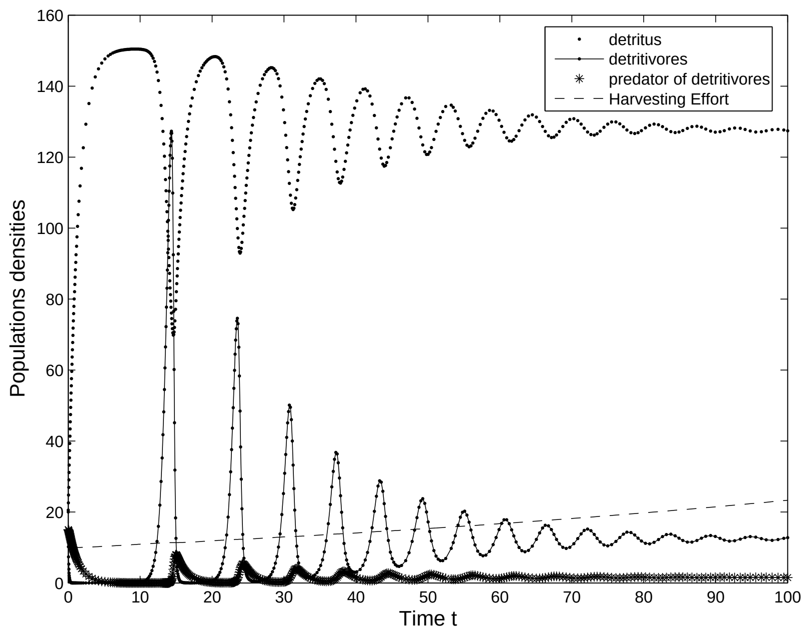

Figure 1: Time series evolution of the populations of the model system (2.1)–(2.4) without time delay

Using the model equations we have

$$\frac{\mathrm{d}V}{\mathrm{d}t} = (x-x^*)\left[\frac{K}{x} - \alpha - \beta y + \frac{d_3 z}{x} - qE\right] + l_1(y-y^*)\left[-d_1 + c_1 x - c_2 z\right]$$

$$+ l_2(z-z^*)\left[-d_2 + c_3 y\right] + l_3(E-E^*)\left[\lambda(p-\sigma)qx - \lambda\gamma\right];$$

$$\frac{\mathrm{d}V}{\mathrm{d}t} = (x-x^*)\left[\left(\frac{K}{x} + \frac{d_3 z}{x}\right) - \left(\frac{K}{x^*} + \frac{d_3 z^*}{x^*}\right)\right] + l_1(y-y^*)\left[(c_1 x - c_2 z) - (c_1 x^* - c_2 z^*)\right]$$

$$+ l_2(z-z^*)\left[c_3 y - c_3 y^*\right] + l_3(E-E^*)\left[(\lambda pqx - \lambda\sigma qx) - (\lambda pqx^* - \lambda\sigma qx^*)\right];$$

$$\frac{\mathrm{d}V}{\mathrm{d}t} = (x-x^*)\left[K\left(\frac{1}{x} - \frac{1}{x^*}\right) + d_3\left(\frac{z}{x} - \frac{z^*}{x^*}\right)\right] + l_1(y-y^*)\left[c_1(x-x^*) - c_2(z-z^*)\right]$$

$$+ l_2(z-z^*)\left[c_3(y-y^*)\right] + l_3(E-E^*)\left[\lambda pq(x-x^*) - \lambda\sigma q(x-x^*)\right];$$

$$\frac{\mathrm{d}V}{\mathrm{d}t} = (x-x^*)\left[K\left(\frac{x^*-x}{xx^*}\right) + d_3\left(\frac{zx^*-z^*x}{xx^*}\right)\right]$$

$$+ l_1\left[c_1(x-x^*)(y-y^*) - c_2(y-y^*)(z-z^*)\right]$$

$$+ l_2\left[c_3(y-y^*)(z-z^*)\right] + l_3(x-x^*)(E-E^*)\left[\lambda pq - \lambda\sigma q\right] \tag{4.15}$$

By choosing $l_1 = \frac{1}{c_2}$; $l_2 = \frac{1}{c_3}$; $l_3 = \frac{1}{(\lambda pq - \lambda\sigma q)}$, we have

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{K(x-x^*)^2}{xx^*} + \frac{d_3}{xx^*}(x-x^*)(zx^* - z^*x)$$

$$+\frac{c_1}{c_2}(x-x^*)(y-y^*)+(x-x^*)(E-E^*);$$

$$\frac{\mathrm{d}V}{\mathrm{d}t}=-\frac{K(x-x^*)^2}{xx^*}-\frac{d_3z^*}{xx^*}(x-x^*)^2+\frac{d_3}{x}(x-x^*)(z-z^*)+\frac{c_1}{c_2}(x-x^*)(y-y^*)$$
$$+(x-x^*)(E-E^*);$$

$$\frac{\mathrm{d}V}{\mathrm{d}t}\le-\frac{K(x-x^*)^2}{xx^*}-\frac{d_3z^*}{xx^*}(x-x^*)^2+\frac{d_3}{x}\left[\frac{(x-x^*)^2}{2}+\frac{(z-z^*)^2}{2}\right]$$
$$+\frac{c_1}{c_2}\left[\frac{(x-x^*)^2}{2}+\frac{(y-y^*)^2}{2}\right]+\frac{(x-x^*)^2}{2}+\frac{(E-E^*)^2}{2}. \quad (4.16)$$

Thus the model (4.1)–(4.4) with $\tau=0$ is globally asymptotically stable.

5 Optimal harvesting policy

In this section we derive an optimal harvesting policy to maximize the total discounted net revenue from the harvesting biomass using the tax σ as a control parameter. The net economic revenue is given by
$\pi(x,y,z,E,\sigma,t)$ = (Net revenue of harvesting agency) $-$ (Net economic revenue to the regulatory agency), that is,

$$\pi(x,y,z,E,\sigma,t)=(p-\sigma)qEx-\gamma E+\sigma qEx=(pqx-\gamma)E. \quad (5.1)$$

Our objective is to maximize the present value function

$$J=\int_0^\infty e^{-\delta t}[pqx-\gamma]\,E(t)\,dt \quad (5.2)$$

where δ is the instantaneous annual rate of discount and the optimization problem is subjected to the model (4.1)–(4.4).
The control variable $\sigma(t)$ is subjected to the constraints $\sigma_{\min}\le\sigma\le\sigma_{\max}$.
Using Pontryagin's maximal principle (1964), the associated Hamiltonian $H(x(t),y(t),z(t),E(t),\sigma(t),t)$ is given by
$H=e^{-\delta t}[pqx-\gamma]\,E+\lambda_1[K-\alpha x-\beta xy+d_3z-qEx]+\lambda_2[-yd_1+yc_1x-yc_2z]$

$$+\lambda_3[-d_2z+c_3yz]+\lambda_4[\lambda pqxE-\lambda\sigma qxE-\lambda\gamma E], \quad (5.3)$$

where $\lambda_i(t)$ $(i=1,2,3,4)$ are adjoint variables.
The condition for a singular control to be optimal is

$$\frac{\partial H}{\partial\sigma}=0\Rightarrow\lambda_4\lambda qxE=0\Rightarrow\lambda_4(t)=0. \quad (5.4)$$

The adjoint equations are

$$\frac{\mathrm{d}\lambda_1}{\mathrm{d}t}=-\frac{\partial H}{\partial x};\ \frac{\mathrm{d}\lambda_2}{\mathrm{d}t}=-\frac{\partial H}{\partial y};\ \frac{\mathrm{d}\lambda_3}{\mathrm{d}t}=-\frac{\partial H}{\partial z};\ \frac{\mathrm{d}\lambda_4}{\mathrm{d}t}=-\frac{\partial H}{\partial E}. \quad (5.5)$$

$$\frac{\mathrm{d}\lambda_1}{\mathrm{d}t}=-\left[e^{-\delta t}pqE+\lambda_1(-\alpha-\beta y-qE)+\lambda_2(yc_1)+\lambda_4(\lambda pqE-\lambda\sigma qE)\right];$$

$$\frac{\mathrm{d}\lambda_2}{\mathrm{d}t} = -\left[\lambda_1(-\beta x) + \lambda_2(-d_1 + c_1 x - c_2 z) + \lambda_3(c_3 z)\right];$$

$$\frac{\mathrm{d}\lambda_3}{\mathrm{d}t} = -\left[\lambda_1 d_3 + \lambda_2(-c_2 y) + \lambda_3(-d_2 + c_3 y)\right];$$

$$\frac{\mathrm{d}\lambda_4}{\mathrm{d}t} = -\left[e^{-\delta t}(pqx - \gamma) + \lambda_1(-qx) + \lambda_4(\lambda pqx - \lambda\sigma qx - \lambda\gamma)\right]. \tag{5.6}$$

When we evaluate these equations at the interior equilibrium point, we get

$$\frac{\mathrm{d}\lambda_1}{\mathrm{d}t} = -e^{-\delta t}pqE - \lambda_1\left(-\frac{K}{x} - \frac{d_3 z}{x}\right) - \lambda_2 c_1 y - \lambda_4(\lambda pqE - \lambda\sigma qE), \tag{5.7}$$

$$\frac{\mathrm{d}\lambda_2}{\mathrm{d}t} = \lambda_1 \beta x - \lambda_3 c_3 z, \tag{5.8}$$

$$\frac{\mathrm{d}\lambda_3}{\mathrm{d}t} = -\lambda_1 d_3 + \lambda_2 c_2 y, \tag{5.9}$$

$$\frac{\mathrm{d}\lambda_4}{\mathrm{d}t} = -e^{-\delta t}(pqx - \gamma) + \lambda_1 qx. \tag{5.10}$$

From (5.10), we get $0 = -e^{-\delta t}(pqx - \gamma) + \lambda_1 qx$, since $\lambda_4(t) = 0$.

This implies that

$$\lambda_1 = e^{-\delta t}\left(p - \frac{\gamma}{qx}\right). \tag{5.11}$$

Therefore, $\frac{\mathrm{d}\lambda_1}{\mathrm{d}t} = -\delta e^{-\delta t}\left(p - \frac{\gamma}{qx}\right)$.

From (5.7), we get, $-\delta e^{-\delta t}\left(p - \frac{\gamma}{qx}\right) = -e^{-\delta t}pqE - e^{-\delta t}\left(p - \frac{\gamma}{qx}\right)\left(-\frac{K}{x} - \frac{d_3 z}{x}\right) - \lambda_2 c_1 y$.

This implies that

$$\lambda_2 = Ae^{-\delta t}, \tag{5.12}$$

where $A = \frac{1}{yc_1}\left[\left(p - \frac{\gamma}{qx}\right)\left(\frac{K}{x} + \frac{d_3 z}{x} + \delta\right) - pqE\right]$. Therefore, $\frac{\mathrm{d}\lambda_2}{\mathrm{d}t} = -\delta Ae^{-\delta t}$.

From (5.8), we get, $-\delta Ae^{-\delta t} = \beta xe^{-\delta t}\left(p - \frac{\gamma}{qx}\right) - \lambda_3 c_3 z$. This implies that

$$\lambda_3 = Be^{-\delta t}, \tag{5.13}$$

where $B = \frac{1}{c_3 z}\left[\beta x\left(p - \frac{\gamma}{qx}\right) + \delta A\right]$. Therefore, $\frac{\mathrm{d}\lambda_3}{\mathrm{d}t} = -\delta Be^{-\delta t}$.
From (5.9), we get $-\delta Be^{-\delta t} = -\left[d_3 e^{-\delta t}\left(p - \frac{\gamma}{qx}\right) + Ae^{-\delta t}(-c_2 y)\right]$. This implies that

$$\left(p - \frac{\gamma}{qx}\right) = \frac{Ac_2 y + \delta B}{d_3}. \tag{5.14}$$

This provides the equation of singular path and gives the optimal equilibrium levels of the populations $x^* = x_\delta$, $y^* = y_\delta$, $z^* = z_\delta$. Then the optimal equilibrium of harvesting effort and tax can be obtained as

$$E_\delta = \frac{1}{qx_\delta}\left[K - x_\delta\left(\alpha + \frac{\beta d_2}{c_3}\right) + d_3 z_\delta\right],$$

$$\sigma_\delta = p - \frac{\gamma}{qx_\delta}. \tag{5.15}$$

From (5.4), (5.11), (5.12) and (5.13), we observe that $\lambda_i(t)e^{\delta\, t}\ (i = 1,\, 2,\, 3,\, 4)$ do not vary with time at the optimal equilibrium. Hence they remain bounded as $t \to \infty$. From (5.11), $\lambda_1(t)\, qx^* = e^{-\delta\, t}(pqx^* - c) = e^{-\delta\, t}\, \frac{\partial\pi}{\partial E}$, which imply that the total users cost of harvest per unit effort is equal to the discounted values of the future price at the interior equilibrium point.

6 Stability analysis with time delay

We now discuss the stability of the model (2.1)–(2.4) in the presence of delay. The Jacobian of the model system (2.1)–(2.4) is

$$\left|\begin{array}{cccc} -\alpha - \beta y - qE & -\beta x & d_3 & -qx \\ c_1 y & -d_1 + c_1 x - c_2 z & -c_2 y & 0 \\ 0 & c_3 z e^{-\mu\,\tau} & -d_2 + c_3 y e^{-\mu\,\tau} & 0 \\ \lambda E(p-\sigma)q & 0 & 0 & \lambda[(p-\sigma)qx - \gamma] \end{array}\right|. \tag{6.1}$$

The characteristic equation of the model system (2.1)–(2.4) at the interior equilibrium is

$$\left|\begin{array}{cccc} -\frac{K}{x} - \frac{d_3 z}{x} - \mu & -\beta x & d_3 & -qx \\ c_1 y & -\mu & -c_2 y & 0 \\ 0 & c_3 z e^{-\mu\,\tau} & -c_3 y + c_3 y e^{-\mu\,\tau} - \mu & 0 \\ \frac{\lambda\gamma E}{x} & 0 & 0 & -\mu \end{array}\right| = 0, \tag{6.2}$$

or,

$$\mu^4 + \mu^3\left(c_3 y + \tfrac{K}{x} + \tfrac{d_3 z}{x}\right) + \mu^2\left(\tfrac{Kc_3 y}{x} + \tfrac{c_3 d_3 yz}{x} + \lambda\gamma Eq + \beta c_1 xy\right) + \mu\left(\lambda\gamma Eqc_3 y + \beta c_1 c_3 xy^2\right)$$
$$+e^{-\mu\tau}\left[\begin{array}{l} \mu^3(-c_3 y) + \mu^2\left(-\tfrac{Kc_3 y}{x} - \tfrac{c_3 d_3 yz}{x} + c_2 c_3 yz\right) \\ +\mu\left(-\lambda\gamma Eqc_3 y + \tfrac{Kc_2 c_3 yz}{x} + \tfrac{c_2 c_3 d_3 yz^2}{x} - \beta c_1 c_3 xy^2 - c_1 c_3 d_3 yz\right) \\ +\lambda\gamma Eqc_2 c_3 yz \end{array}\right] = 0. \tag{6.3}$$

(6.3) is in the form of

$$X(\mu) + e^{-\mu\,\tau} Y(\mu) = 0, \tag{6.4}$$

where $X(\mu) = \mu^4 + x_1\mu^3 + x_2\mu^2 + x_3\mu$, $Y(\mu) = y_1\mu^3 + y_2\mu^2 + y_3\mu + y_4$,

$x_1 = c_3 y + \frac{K}{x} + \frac{d_3 z}{x}$, $x_2 = \frac{Kc_3 y}{x} + \frac{c_3 d_3 yz}{x} + \lambda\gamma Eq + \beta c_1 xy$, $x_3 = \lambda\gamma Eqc_3 y + \beta c_1 c_3 xy^2$,

$y_1 = -c_3 y$, $y_2 = -\frac{Kc_3 y}{x} - \frac{c_3 d_3 yz}{x} + c_2 c_3 yz$,

$y_3 = -\lambda\gamma Eqc_3 y + \frac{Kc_2 c_3 yz}{x} + \frac{c_2 c_3 d_3 yz^2}{x} - \beta c_1 c_3 xy^2 - c_1 c_3 d_3 yz$, $y_4 = \lambda\gamma Eqc_2 c_3 yz$.

Let $\mu = i\omega$ be a root of (6.1), where ω is a real number. Putting $\mu = i\omega$ in (6.1), we get

$$\omega^4 - i\omega^3 x_1 - \omega^2 x_2 + i\omega x_3 + (\cos\omega\tau - i\sin\omega\tau)(-i\omega^3 y_1 - \omega^2 y_2 + i\omega y_3 + y_4) = 0. \tag{6.5}$$

Separating the real and imaginary parts, we get

$$\omega^4 - \omega^2 x_2 = (\omega^2 y_2 - y_4)\cos\omega\tau + (\omega^3 y_1 - \omega y_3)\sin\omega\tau, \tag{6.6}$$

$$\omega^3 x_1 - \omega x_3 = (\omega^2 y_2 - y_4)\sin\omega\tau - (\omega^3 y_1 - \omega y_3)\cos\omega\tau. \tag{6.7}$$

Squaring and adding (6.5) and (6.6), we get

$$\left(\omega^4 - \omega^2 x_2\right)^2 + \left(\omega^3 x_1 - \omega x_3\right)^2 = (\omega^2 y_2 - y_4)^2 + (\omega^3 y_1 - \omega y_3)^2,$$

or,

$$\omega^8 + \omega^6(-2x_2 + x_1^2 - y_1^2) + \omega^4(x_2^2 - 2x_1x_3 - y_2^2 + 2y_1y_3) + \omega^2\left(x_3^2 + 2y_2y_4 - y_3^2\right) - y_4^2 = 0. \tag{6.8}$$

Equation (6.8) is in the form of

$$\omega^8 + B_1\omega^6 + B_2\omega^4 + B_3\omega^2 + B_4 = 0, \tag{6.9}$$

where $B_1 = -2x_2 + x_1^2 - y_1^2$, $B_2 = x_2^2 - 2x_1x_3 - y_2^2 + 2y_1y_3$, $B_3 = x_3^2 + 2y_2y_4 - y_3^2$, $B_4 = -y_4^2$.

By Descartes' rule, if (i) $B_1 > 0, B_2 > 0$ and $B_3 > 0$; or, (ii) $B_1 > 0, B_2 > 0$ and $B_3 < 0$; or, (iii) $B_1 > 0, B_2 < 0$, and $B_3 < 0$, then (6.9) has a unique positive root, ω_0 (say), and then has a pair of imaginary roots $\pm i\,\omega_0$. Eliminating $\sin\omega\tau$ from (6.2) and (6.3), we get

$$\cos\omega\tau = \frac{\left(\omega^4 - \omega^2 x_2\right)\left(\omega^2 y_2 - y_4\right) - \left(\omega^3 y_1 - \omega y_3\right)\left(\omega^3 x_1 - \omega x_3\right)}{\left(\omega^3 y_1 - \omega y_3\right)^2 + \left(\omega^2 y_2 - y_4\right)^2}. \tag{6.10}$$

Then τ_k corresponding to $\omega = \omega_0$ is given by

$$\tau_k = \frac{1}{\omega_0}\cos^{-1}\left[\frac{\left(\omega_0^4 - \omega_0^2 x_2\right)\left(\omega_0^2 y_2 - y_4\right) - \left(\omega_0^3 y_1 - \omega_0 y_3\right)\left(\omega_0^3 x_1 - \omega_0 x_3\right)}{\left(\omega_0^3 y_1 - \omega_0 y_3\right)^2 + \left(\omega_0^2 y_2 - y_4\right)^2}\right] + \frac{2k\pi}{\omega_0},\ k = 0,\ 1,\ 2,\ 3, \ldots \tag{6.11}$$

By Butler's lemma, we conclude that the model (2.1)–(2.4) is stable around the interior equilibrium for $\tau < \tau_0$ as $k = 0$.

Now differentiating the characteristic equation (6.1) with respect to τ, we get

$$X'(\mu)\frac{\mathrm{d}\mu}{\mathrm{d}\tau} + e^{-\mu\tau}Y'(\mu)\frac{\mathrm{d}\mu}{\mathrm{d}\tau} + Y(\mu)e^{-\mu\tau}\left(-\mu - \tau\frac{\mathrm{d}\mu}{\mathrm{d}\tau}\right) = 0. \tag{6.12}$$

Or,

$$\begin{aligned}\left(\frac{\mathrm{d}\mu}{\mathrm{d}\tau}\right)^{-1} &= \frac{X'(\mu)}{-\mu X(\mu)} + \frac{Y'(\mu)}{\mu Y(\mu)} - \frac{\tau}{\mu} \\ &= \frac{4\mu^3 + 3x_1\mu^2 + 2x_2\mu + x_3}{-\mu(\mu^4 + x_1\mu^3 + x_2\mu^2 + x_3\mu)} + \frac{3y_1\mu^2 + 2y_2\mu + y_3}{\mu(y_1\mu^3 + y_2\mu^2 + y_3\mu + y_4)} - \frac{\tau}{\mu} \\ &= \frac{4\mu^4 + 3x_1\mu^3 + 2x_2\mu^2 + x_3\mu}{-\mu^2(\mu^4 + x_1\mu^3 + x_2\mu^2 + x_3\mu)} + \frac{3y_1\mu^3 + 2y_2\mu^2 + y_3\mu}{\mu^2(y_1\mu^3 + y_2\mu^2 + y_3\mu + y_4)} - \frac{\tau}{\mu} \\ &= \frac{3\mu^4 + 2x_1\mu^3 + x_2\mu^2}{-\mu^2(\mu^4 + x_1\mu^3 + x_2\mu^2 + x_3\mu)} + \frac{2y_1\mu^3 + y_2\mu^2 - y_4}{\mu^2(y_1\mu^3 + y_2\mu^2 + y_3\mu + y_4)} - \frac{\tau}{\mu}.\end{aligned} \tag{6.13}$$

Thus

$$\left[\left(\frac{\mathrm{d}\mu}{\mathrm{d}\tau}\right)^{-1}\right]_{\mu = i\omega_0} = \frac{(3\omega_0^4 - x_2\omega_0^2) - 2ix_1\omega_0^3}{\omega_0^2\left[(\omega_0^4 - \omega_0^2 x_2) - i(\omega_0^3 x_1 - \omega_0 x_3)\right]}$$

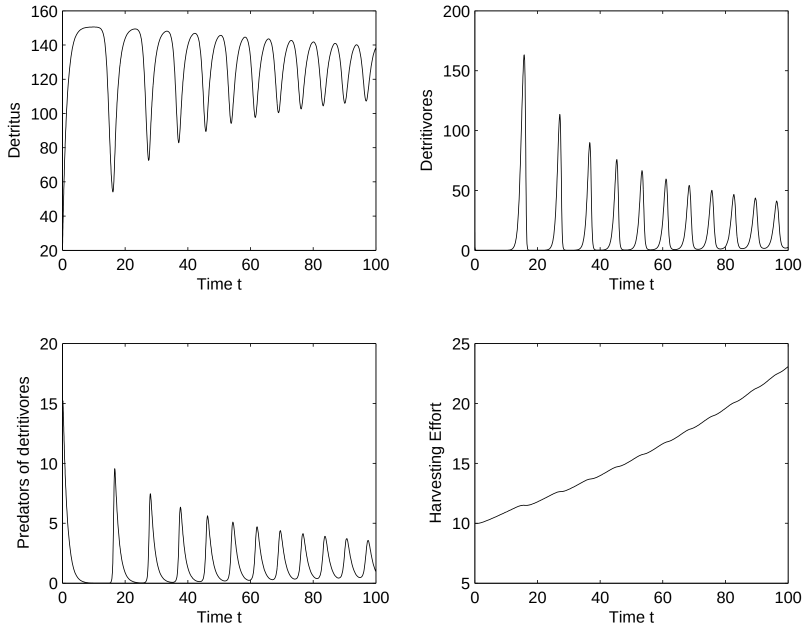

Figure 2: Stable population densities of the delay model system (2.1)–(2.4) with $\tau = 0.1$

$$+\frac{(\omega_0^2 y_2 + y_4) + 2iy_1\omega_0^3}{\omega_0^2\left[-(\omega_0^2 y_2 - y_4) - i(\omega_0^3 y_1 - \omega_0 y_3)\right]} - \frac{\tau}{i\omega_0} \tag{6.14}$$

$$\mathrm{Re}\left[\left(\frac{\mathrm{d}\mu}{\mathrm{d}\tau}\right)^{-1}\right]_{\mu=i\omega_0} = \frac{1}{\omega_0^2}\Big[\frac{(3\omega_0^4 - x_2\omega_0^2)(\omega_0^4 - \omega_0^2 x_2) + 2x_1\omega_0^3(\omega_0^3 x_1 - \omega_0 x_3)}{(\omega_0^4 - \omega_0^2 x_2)^2 + (\omega_0^3 x_1 - \omega_0 x_3)^2}$$

$$+\frac{(y_4 + \omega_0^2 y_2)(y_4 - \omega_0^2 y_2) + 2y_1\omega_0^3(\omega_0 y_3 - \omega_0^3 y_1)}{(y_4 - \omega_0^2 y_2)^2 + (\omega_0^3 y_1 - \omega_0 y_3)^2}\Big] \tag{6.15}$$

$$= \frac{1}{\omega_0^2\,\xi^2}\left[3\omega_0^8 + 2(x_1^2 - 2\,x_2 - y_1^2)\,\omega_0^6 + (x_2^2 - 2x_1x_3 + 2y_1y_3 - y_2^2)\,\omega_0^4 + y_4^2\right] \tag{6.16}$$

$$= \frac{1}{\omega_0^2\,\xi^2}\left[2\omega_0^8 + (x_1^2 - 2\,x_2 - y_1^2)\,\omega_0^6 - (x_3 - y_3^2 - 2x_2x_4 + 2y_2y_4)\,\omega_0^2 - 2(-y_4^2)\right], \tag{6.17}$$

(using (6.9)) where

$$\xi^2 = (\omega_0^4 - \omega_0^2 x_2)^2 + (\omega_0^3 x_1 - \omega_0 x_3)^2 = (y_4 - \omega_0^2 y_2)^2 + (\omega_0^3 y_1 - \omega_0 y_3)^2.$$

Now on the basis of existence of positive root of (6.9), Sign of Re $\left[\left(\frac{\mathrm{d}\mu}{\mathrm{d}\tau}\right)^{-1}\right]$ is positive if $x_1^2 - 2\,x_2 - y_1^2 > 0,$, i.e., if

$$\frac{K^2}{x^2} + \frac{d_3^2 z^2}{x^2} + \frac{2Kd_3 z}{x^2} - 2\lambda\gamma Eq - 2\beta c_1 xy > 0 \tag{6.18}$$

when evaluated at the interior equilibrium point G_3. Therefore,

$$sign\ \left(\frac{\mathrm{d}}{\mathrm{d}\tau}(\,\mathrm{Re}\,\mu\,)\right) = sign\ \left\{\mathrm{Re}\ \left(\frac{\mathrm{d}\mu}{\mathrm{d}\tau}\right)^{-1}\right\}.$$

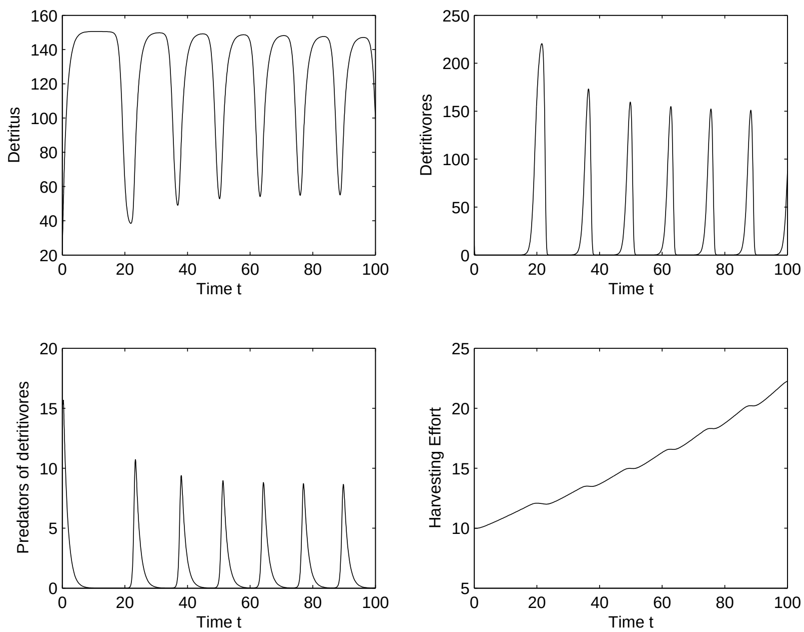

Figure 3: Periodic oscillation of the of populations of the model system (2.1)–(2.4) with $\tau = 0.3419$

Thus, the transversality condition holds and Hopf - bifurcation occurs at $\tau = \tau_0$.

In this present paper, we assume the presence of randomly fluctuating driving forces on the deterministic growth of the prey, predator-1 and predator-2 populations at time t, so that the system (2.1)–(2.4) results in the stochastic delay system with additive noise.

7 The stochastic delayed model

The main assumption that leads us to extend the deterministic model (2.1)–(2.4) to a stochastic counterpart is that it is reasonable to conceive the open sea as a noisy environment. There are many number of ways in which the environmental noise may be incorporated in the system (2.1)–(2.4). Note that environmental noise should be distinguished from a demographic or internal noise, for which the variation over time is due. External noise may arise either form random fluctuations of one or more model-parameters around some known mean values or from stochastic fluctuations of the population densities around some constant values. In this section, we compute the population intensities of fluctuations (variances) around the positive equilibrium G_3 due to noise, according to the method introduced by [34, 20, 1]. Such a method was also successfully applied in [23, 2]. Now we assume the presence of randomly fluctuating driving forces on the deterministic growth of the prey, predator-1 and predator-2 populations at time t, so that the system (2.1)–(2.4) results in a stochastic delay system with additive noise.

$$\frac{\mathrm{d}x}{\mathrm{d}t} = K - \alpha x - \beta xy + d_3 z - qEx + \eta_1 \xi_1(t), \tag{7.1}$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y(-d_1 + c_1 x - c_2 z) + \eta_2 \xi_2(t), \tag{7.2}$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = -d_2 z + c_3 y(t-\tau) z(t-\tau) + \eta_3 \xi_3(t), \tag{7.3}$$

$$\frac{dE}{dt} = \lambda \left[(p-\sigma) q x - \gamma \right] E + \eta_4 \xi_4(t), \tag{7.4}$$

where $x(t)$ represents density of biomass of the plant litter of the mangroves plants after decomposition (detritus) at time t, $y(t)$ represents biomass of micro organisms (detritivorus) at time t, $z(t)$ represents biomass density of predators of detritivorus at time t, $E = E(t)$ represents harvesting effort at any time t and $\xi(t) = [\xi_1(t), \xi_2(t), \xi_3(t), \xi_4(t)]$ is a 4D Gaussian white noise process satisfying

$$E\left[\xi_i(t)\right] = 0;\ i = 1, 2, 3, 4, \tag{7.5}$$

$$E\left[\xi_i(t)\, \xi_j(t')\right] = \delta_{ij}\, \delta\left(t - t'\right);\ i, j = 1, 2, 3, 4 \tag{7.6}$$

where δ_{ij} is the Kronecker symbol δ is the Dirac-delta function. Let

$$x(t) = u_1(t) + S^*;\ y(t) = u_2(t) + P^*;\ z(t) = u_3(t) + T^*;\ E(t) = u_4(t) + U^*; \tag{7.7}$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{du_1(t)}{dt}; \frac{\mathrm{d}y}{\mathrm{d}t} = \frac{du_2(t)}{dt}; \frac{\mathrm{d}z}{\mathrm{d}t} = \frac{du_3(t)}{dt}; \frac{dE}{dt} = \frac{du_4(t)}{dt}; \tag{7.8}$$

Using (7.7) and (7.8), equation (7.1) becomes

$$\begin{aligned} \frac{du_1(t)}{dt} &= K - \alpha u_1(t) - \alpha S^* - \beta u_1(t) u_2(t) - \beta u_1(t) P^* - \beta u_2(t) S^* - \beta S^* P^* \\ &\quad + d_3 u_3(t) + d_3 T^* - q u_1(t) u_4(t) - q u_4(t) S^* - q u_1(t) U^* - q S^* U^* + \eta_1 \xi_1(t) \end{aligned} \tag{7.9}$$

The linear part of (7.9) is

$$\frac{du_1(t)}{dt} = -\beta u_2(t) S^* - q u_4(t) S^* + \eta_1 \xi_1(t). \tag{7.10}$$

Using (7.7) and (7.8), equation (7.2) becomes

$$\begin{aligned} \frac{du_2(t)}{dt} &= -d_1 u_2(t) - d_1 P^* + c_1 u_1(t) u_2(t) + c_1 u_1(t) P^* + c_1 u_2(t) S^* \\ &\quad + c_1 S^* P^* - c_2 u_2(t) u_3(t) - c_2 u_3(t) P^* - c_2 u_2(t) T^* - c_2 P^* T^* + \eta_2 \xi_2(t) \end{aligned} \tag{7.11}$$

The linear part of (7.11) is

$$\frac{du_2(t)}{dt} = c_1 u_1(t) P^* - c_2 u_3(t) P^* + \eta_2 \xi_2(t). \tag{7.12}$$

Using (7.7) and (7.8), equation (7.3) becomes

$$\begin{aligned} \frac{du_3(t)}{dt} &= -d_2 u_3(t) - d_2 T^* + c_3 u_2(t-\tau) u_3(t-\tau) + c_3 u_2(t-\tau) T^* \\ &\quad + c_3 u_3(t-\tau) P^* + c_3 P^* T^* + \eta_3 \xi_3(t) \end{aligned} \tag{7.13}$$

The linear part of (7.13) is

$$\frac{du_3(t)}{dt} = c_3 u_2(t-\tau) T^* + \eta_3 \xi_3(t). \tag{7.14}$$

Using (7.7) and (7.8), equation (7.4) becomes

$$\begin{aligned} \frac{du_4(t)}{dt} &= \lambda(p-\sigma) q u_1(t) u_4(t) + \lambda(p-\sigma) q u_4(t) S^* - \lambda \gamma u_4(t) \\ &\quad + \lambda(p-\sigma) q u_1(t) U^* + \lambda(p-\sigma) q S^* U^* - \lambda \gamma U^* + + \eta_4 \xi_4(t) \end{aligned}\ . \tag{7.15}$$

The linear part of (7.15) is

$$\frac{du_4(t)}{dt} = \lambda(p-\sigma)qu_1(t)U^* + \eta_4\xi_4(t). \tag{7.16}$$

Taking the Fourier transform on both sides of (7.10), (7.12), (7.14) and (7.16), we get

$$i\omega\tilde{u}_1(\omega) = -\beta S^*\tilde{u}_2(\omega) - qS^*\tilde{u}_4(\omega) + \eta_1\tilde{\xi}_1(\omega)$$

$$(or)\, \eta_1\tilde{\xi}_1(\omega) = i\omega\tilde{u}_1(\omega) + \beta S^*\tilde{u}_2(\omega) + qS^*\tilde{u}_4(\omega) \tag{7.17}$$

$$i\omega\tilde{u}_2(\omega) = c_1P^*\tilde{u}_1(\omega) - c_2P^*\tilde{u}_3(\omega) + \eta_2\tilde{\xi}_2(\omega)$$

$$(or)\, \eta_2\tilde{\xi}_2(\omega) = -c_1P^*\tilde{u}_1(\omega) + i\omega\tilde{u}_2(\omega) + c_2P^*\tilde{u}_3(\omega) \tag{7.18}$$

$$i\omega\tilde{u}_3(\omega) = c_3T^*e^{-i\omega\tau}\tilde{u}_2(\omega) + \eta_3\tilde{\xi}_3(\omega)$$

$$(or)\, \eta_3\tilde{\xi}_3(\omega) = -c_3T^*e^{-i\omega\tau}\tilde{u}_2(\omega) + i\omega\tilde{u}_3(\omega) \tag{7.19}$$

$$i\omega\tilde{u}_4(\omega) = \lambda(p-\sigma)q\tilde{u}_1(\omega)U^* + \eta_4\tilde{\xi}_4(\omega)$$

$$(or)\, \eta_4\tilde{\xi}_4(\omega) = -\lambda(p-\sigma)q\tilde{u}_1(\omega)U^* + i\omega\tilde{u}_4(\omega). \tag{7.20}$$

The matrix form of (7.17), (7.18), (7.19) and (7.20) is

$$M(\omega)\,\tilde{u}(\omega) = \tilde{\xi}(\omega) \tag{7.21}$$

where

$$M(\omega) = \begin{pmatrix} A_{11}(\omega) & A_{12}(\omega) & A_{13}(\omega) & A_{14}(\omega) \\ A_{21}(\omega) & A_{22}(\omega) & A_{23}(\omega) & A_{24}(\omega) \\ A_{31}(\omega) & A_{32}(\omega) & A_{33}(\omega) & A_{34}(\omega) \\ A_{41}(\omega) & A_{42}(\omega) & A_{43}(\omega) & A_{44}(\omega) \end{pmatrix};$$

$$\tilde{u}(\omega) = \begin{bmatrix} \tilde{u}_1(\omega) \\ \tilde{u}_2(\omega) \\ \tilde{u}_3(\omega) \\ \tilde{u}_4(\omega) \end{bmatrix};\ \tilde{\xi}(\omega) = \begin{bmatrix} \eta_1\tilde{\xi}_1(\omega) \\ \eta_2\tilde{\xi}_2(\omega) \\ \eta_3\tilde{\xi}_3(\omega) \\ \eta_4\tilde{\xi}_4(\omega) \end{bmatrix};$$

$$\begin{array}{l} A_{11}(\omega) = i\omega;\ A_{12}(\omega) = \beta S^*;\ A_{13}(\omega) = 0;\ A_{14}(\omega) = qS^*; \\ A_{21}(\omega) = -c_1P^*;\ A_{22}(\omega) = i\omega;\ A_{23}(\omega) = c_2P^*;\ A_{24}(\omega) = 0; \\ A_{31}(\omega) = 0;\ A_{32}(\omega) = -c_3T^*e^{-i\omega\tau};\ A_{33}(\omega) = i\omega;\ A_{34}(\omega) = 0; \\ A_{41}(\omega) = -\lambda(p-\sigma)qU^*; A_{42}(\omega) = 0; A_{43}(\omega) = 0; A_{44}(\omega) = i\omega. \end{array} \tag{7.22}$$

Equation (7.21) can also be written as $\tilde{u}(\omega) = [M(\omega)]^{-1}\tilde{\xi}(\omega)$. Let $[M(\omega)]^{-1} = K(\omega)$, then

$$\tilde{u}(\omega) = K(\omega)\tilde{\xi}(\omega), \tag{7.23}$$

where

$$K(\omega) = \frac{Adj\, M(\omega)}{|M(\omega)|}. \tag{7.24}$$

If the function $Y(t)$ has a zero mean value, then the fluctuation intensity (variance) of it's components in the frequency interval $[\omega, \omega + d\omega]$ is , where $S_Y(\omega)$ is spectral density of Y and is defined as

$$S_Y(\omega) = \lim_{\tilde{T}\to\infty} \frac{\left|\tilde{Y}(\omega)\right|^2}{\tilde{T}}. \tag{7.25}$$

If Y has a zero mean value, the inverse transform of $S_Y(\omega)$is the auto covariance function

$$C_Y(\tau) = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} S_Y(\omega)e^{i\omega\tau}d\omega. \tag{7.26}$$

The corresponding variance of fluctuations in $Y(t)$ is given by

$$\sigma_Y^2 = C_Y(0) = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} S_Y(\omega)d\omega \tag{7.27}$$

and the auto correlation function is the normalized auto covariance

$$P_Y(\tau) = \frac{C_Y(\tau)}{C_Y(0)}. \tag{7.28}$$

For a Gaussian white noise process, it is

$$\begin{aligned} S_{\xi_i\xi_j}(\omega) &= \lim_{\hat{T}\to+\infty} \frac{E\left[\tilde{\xi}_i(\omega)\tilde{\xi}_j(\omega)\right]}{\hat{T}} \\ &= \lim_{\hat{T}\to+\infty} \frac{1}{\hat{T}} \int\limits_{-\frac{\hat{T}}{2}}^{\frac{\hat{T}}{2}} \int\limits_{-\frac{\hat{T}}{2}}^{\frac{\hat{T}}{2}} E\left[\tilde{\xi}_i(t)\,\tilde{\xi}_j(t')\right] e^{-i\omega(t-t')}dt\,dt' \\ &= \delta_{ij}. \end{aligned} \tag{7.29}$$

From (7.23), we have

$$\tilde{u}_i(\omega) = \sum_{j=1}^{4} K_{ij}(\omega)\,\tilde{\xi}_j(\omega), i = 1,2,3,4. \tag{7.30}$$

From (7.25) we have

$$S_{u_i}(\omega) = \sum_{j=1}^{4} \eta_j\,|K_{ij}(\omega)|^2, \quad i = 1,2,3,\,4. \tag{7.31}$$

Hence by (7.27) and (7.31), the intensities of fluctuations in the variables u_i, $i = 1,2,3,\,4$, are given by

$$\sigma_{u_i}^2 = \frac{1}{2\pi}\sum_{j=1}^{4}\int\limits_{-\infty}^{\infty} \eta_j\,|K_{ij}(\omega)|^2\,d\omega;\;\; i = 1,2,3,\,4 \tag{7.32}$$

and by (7.24), we obtain

$$\begin{aligned}
\sigma_{u_1}^2 &= \tfrac{1}{2\pi}\left\{\int\limits_{-\infty}^{\infty}\eta_1\left|\tfrac{Adj(1)}{|M(\omega)|}\right|^2 d\omega + \int\limits_{-\infty}^{\infty}\eta_2\left|\tfrac{Adj(2)}{|M(\omega)|}\right|^2 d\omega + \int\limits_{-\infty}^{\infty}\eta_3\left|\tfrac{Adj(3)}{|M(\omega)|}\right|^2 d\omega + \int\limits_{-\infty}^{\infty}\eta_4\left|\tfrac{Adj(4)}{|M(\omega)|}\right|^2 d\omega\right\}, \\
\sigma_{u_2}^2 &= \tfrac{1}{2\pi}\left\{\int\limits_{-\infty}^{\infty}\eta_1\left|\tfrac{Adj(5)}{|M(\omega)|}\right|^2 d\omega + \int\limits_{-\infty}^{\infty}\eta_2\left|\tfrac{Adj(6)}{|M(\omega)|}\right|^2 d\omega + \int\limits_{-\infty}^{\infty}\eta_3\left|\tfrac{Adj(7)}{|M(\omega)|}\right|^2 d\omega + \int\limits_{-\infty}^{\infty}\eta_4\left|\tfrac{Adj(8)}{|M(\omega)|}\right|^2 d\omega\right\}, \\
\sigma_{u_3}^2 &= \tfrac{1}{2\pi}\left\{\int\limits_{-\infty}^{\infty}\eta_1\left|\tfrac{Adj(9)}{|M(\omega)|}\right|^2 d\omega + \int\limits_{-\infty}^{\infty}\eta_2\left|\tfrac{Adj(10)}{|M(\omega)|}\right|^2 d\omega + \int\limits_{-\infty}^{\infty}\eta_3\left|\tfrac{Adj(11)}{|M(\omega)|}\right|^2 d\omega + \int\limits_{-\infty}^{\infty}\eta_4\left|\tfrac{Adj(12)}{|M(\omega)|}\right|^2 d\omega\right\}, \\
\sigma_{u_4}^2 &= \tfrac{1}{2\pi}\left\{\int\limits_{-\infty}^{\infty}\eta_1\left|\tfrac{Adj(13)}{|M(\omega)|}\right|^2 d\omega + \int\limits_{-\infty}^{\infty}\eta_2\left|\tfrac{Adj(14)}{|M(\omega)|}\right|^2 d\omega + \int\limits_{-\infty}^{\infty}\eta_3\left|\tfrac{Adj(15)}{|M(\omega)|}\right|^2 d\omega + \int\limits_{-\infty}^{\infty}\eta_4\left|\tfrac{Adj(16)}{|M(\omega)|}\right|^2 d\omega\right\},
\end{aligned} \tag{7.33}$$

where $|M(\omega)| = R(\omega) + iI(\omega)$,

$$R(\omega) = \omega^4 - \omega^2\lambda(p-\sigma)q^2S^*U^* - \omega^2 c_2c_3P^*T^*\cos\omega\tau + \lambda(p-\sigma)q^2c_2c_3S^*P^*T^*U^*\cos\omega\tau, \tag{7.34}$$

$$I(\omega) = -\lambda(p-\sigma)q^2c_2c_3S^*P^*T^*U^*\sin\omega\tau + \omega^2c_2c_3P^*T^*\sin\omega\tau. \tag{7.35}$$

The adjoints are given by

$$|Adj(k)|^2 = X_k^2 + Y_k^2, \quad k = 1,2,3,...,16\,;$$

where

$X_1 = \omega c_2c_3P^*T^*\sin\omega\tau;\ Y_1 = -\omega^3 + \omega c_2c_3P^*T^*\cos\omega\tau;$
$X_2 = -\omega^2c_1P^*;\ Y_2 = 0\,;$
$X_3 = \omega c_1c_3P^*T^*\sin\omega\tau\,;\ Y_3 = \omega c_1c_3P^*T^*\cos\omega\tau\,;$
$X_4 = -\omega^2\lambda(p-\sigma)qU^* + \lambda(p-\sigma)qc_2c_3P^*T^*U^*\cos\omega\tau\,;\ Y_4 = -\lambda(p-\sigma)qc_2c_3P^*T^*U^*\sin\omega\tau\,;$
$X_5 = \omega^2\beta S^*\,; Y_5 = 0; X_6 = 0\,; Y_6 = -\omega^3;$
$X_7 = -\omega^2c_3T^*\cos\omega\tau\,; Y_7 = \omega^2c_3T^*\sin\omega\tau;$
$X_8 = 0; Y_8 = -\omega\lambda(p-\sigma)q\beta S^*U^*;$
$X_9 = 0\,; Y_9 = \omega\beta c_2S^*P^*\,;$
$X_{10} = \omega^2c_2P* - \lambda(p-\sigma)q^2c_2S^*P^*U^*\,; Y_{10} = 0\,;$
$X_{11} = 0; Y_{11} = -\omega^3 + \omega\beta c_1S^*P^* + \omega\lambda(p-\sigma)q^2S^*U^*;$
$X_{12} = \lambda(p-\sigma)q\beta c_2S^*P^*U^*\,; Y_{12} = 0\,;$
$X_{13} = \omega^2qS^* - qc_2c_3S^*P^*T^*\cos\omega\tau\,; Y_{13} = qc_2c_3S^*P^*T^*\sin\omega\tau\,;$
$X_{14} = 0; Y_{14} = -\omega qc_1S^*P^*\,;$
$X_{15} = -qc_1c_3S^*P^*T^*\cos\omega\tau\,; Y_{15} = qc_1c_3S^*P^*T^*\sin\omega\tau\,;$
$X_{16} = \omega c_2c_3P^*T^*\sin\omega\tau; Y_{16} = -\omega^3 + \omega c_2c_3P^*T^*\cos\omega\tau + \omega c_1\beta S^*P^*.$

Thus (7.33) becomes

$$\sigma_{u_1}^2 = \frac{1}{2\pi}\left\{\int_{-\infty}^{\infty}\frac{1}{R^2(\omega)+I^2(\omega)}\left[\eta_1\left(X_1^2+Y_1^2\right)+\eta_2\left(X_2^2+Y_2^2\right)+\eta_3\left(X_3^2+Y_3^2\right)+\eta_4\left(X_4^2+Y_4^2\right)\right]d\omega\right\},$$
$$\sigma_{u_2}^2 = \frac{1}{2\pi}\left\{\int_{-\infty}^{\infty}\frac{1}{R^2(\omega)+I^2(\omega)}\left[\eta_1\left(X_5^2+Y_5^2\right)+\eta_2\left(X_6^2+Y_6^2\right)+\eta_3\left(X_7^2+Y_7^2\right)+\eta_4\left(X_8^2+Y_8^2\right)\right]d\omega\right\},$$
$$\sigma_{u_3}^2 = \frac{1}{2\pi}\left\{\int_{-\infty}^{\infty}\frac{1}{R^2(\omega)+I^2(\omega)}\left[\eta_1\left(X_9^2+Y_9^2\right)+\eta_2\left(X_{10}^2+Y_{10}^2\right)+\eta_3\left(X_{11}^2+Y_{11}^2\right)+\eta_4\left(X_{12}^2+Y_{12}^2\right)\right]d\omega\right\},$$
$$\sigma_{u_4}^2 = \frac{1}{2\pi}\left\{\int_{-\infty}^{\infty}\frac{1}{R^2(\omega)+I^2(\omega)}\left[\eta_1\left(X_{13}^2+Y_{13}^2\right)+\eta_2\left(X_{14}^2+Y_{14}^2\right)+\eta_3\left(X_{15}^2+Y_{15}^2\right)+\eta_4\left(X_{16}^2+Y_{16}^2\right)\right]d\omega\right\}.$$

If we are interested in the dynamics of system (7.1)–(7.4) with $\eta_1 = \eta_2 = \eta_3 = 0$, then the population variances are:

$$\sigma_{u_1}^2 = \frac{\eta_4}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_4^2+Y_4^2\right)}{R^2(\omega)+I^2(\omega)}d\omega\,,\quad \sigma_{u_2}^2 = \frac{\eta_4}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_8^2+Y_8^2\right)}{R^2(\omega)+I^2(\omega)}d\omega\,,$$
$$\sigma_{u_3}^2 = \frac{\eta_4}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_{12}^2+Y_{12}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega\,,\quad \sigma_{u_4}^2 = \frac{\eta_4}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_{16}^2+Y_{16}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega\,.$$

If $\eta_1 = \eta_2 = \eta_4 = 0$, then the variances are

$$\sigma_{u_1}^2 = \frac{\eta_3}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_3^2+Y_3^2\right)}{R^2(\omega)+I^2(\omega)}d\omega\,,\quad \sigma_{u_2}^2 = \frac{\eta_3}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_7^2+Y_7^2\right)}{R^2(\omega)+I^2(\omega)}d\omega\,,$$
$$\sigma_{u_3}^2 = \frac{\eta_3}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_{11}^2+Y_{11}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega\,,\quad \sigma_{u_4}^2 = \frac{\eta_3}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_{15}^2+Y_{15}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega\,.$$

If $\eta_1 = \eta_3 = \eta_4 = 0$, then the variances are

$$\sigma^2_{u_1} = \frac{\eta_2}{2\pi} \int\limits_{-\infty}^{\infty} \frac{\left(X_2^2+Y_2^2\right)}{R^2(\omega)+I^2(\omega)}\, d\omega\ ,\ \ \sigma^2_{u_2} = \frac{\eta_2}{2\pi} \int\limits_{-\infty}^{\infty} \frac{\left(X_6^2+Y_6^2\right)}{R^2(\omega)+I^2(\omega)}\, d\omega\ ,$$
$$\sigma^2_{u_3} = \frac{\eta_2}{2\pi} \int\limits_{-\infty}^{\infty} \frac{\left(X_{10}^2+Y_{10}^2\right)}{R^2(\omega)+I^2(\omega)}\, d\omega\ ,\ \ \sigma^2_{u_4} = \frac{\eta_2}{2\pi} \int\limits_{-\infty}^{\infty} \frac{\left(X_{14}^2+Y_{14}^2\right)}{R^2(\omega)+I^2(\omega)}\, d\omega\ .$$

If $\eta_2 = \eta_3 = \eta_4 = 0$, then

$$\sigma^2_{u_1} = \frac{\eta_1}{2\pi} \int\limits_{-\infty}^{\infty} \frac{\left(X_1^2+Y_1^2\right)}{R^2(\omega)+I^2(\omega)}\, d\omega\ ,\ \ \sigma^2_{u_2} = \frac{\eta_1}{2\pi} \int\limits_{-\infty}^{\infty} \frac{\left(X_5^2+Y_5^2\right)}{R^2(\omega)+I^2(\omega)}\, d\omega\ ,$$
$$\sigma^2_{u_2} = \frac{\eta_1}{2\pi} \int\limits_{-\infty}^{\infty} \frac{\left(X_9^2+Y_9^2\right)}{R^2(\omega)+I^2(\omega)}\, d\omega\ ,\ \ \sigma^2_{u_4} = \frac{\eta_1}{2\pi} \int\limits_{-\infty}^{\infty} \frac{\left(X_{13}^2+Y_{13}^2\right)}{R^2(\omega)+I^2(\omega)}\, d\omega\ .$$

The four expressions in (7.33) can be evaluated numerically which gives the variances of the populations.

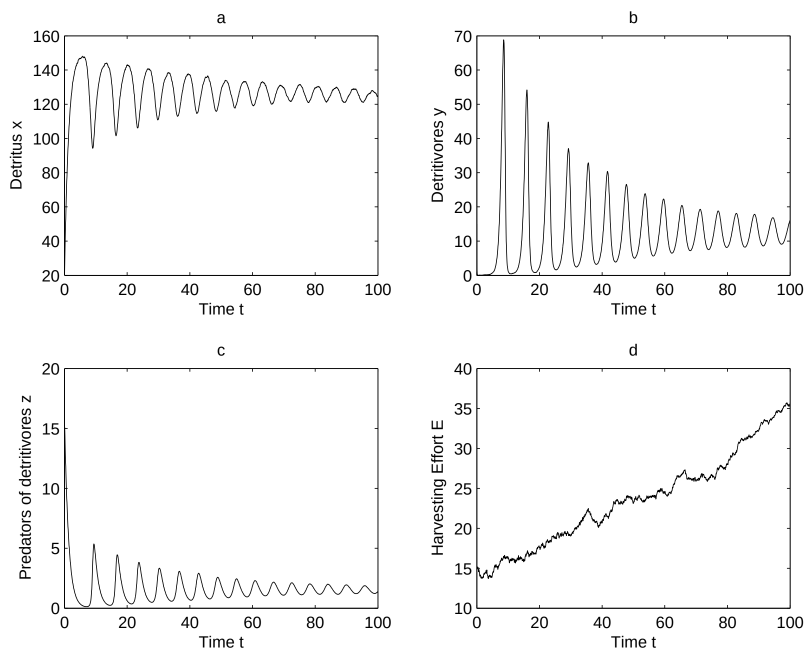

Figure 4: Stable oscillations of stochastic model system (7.1)–(7.4) with small intensity of the noise

8 Conclusion

The importance of sustainable development has gained focus in the recent times and idea seems to resonate with so many people. Without immediate and effective action, our universal planet will face unyielding pressure on the environment. This is the high time when action is needed and it will be guided through a scientific understanding of the ecosystems function. Ecological modelling supports the sustainable development paradigm where economy, society and environment are integrated positively reinforcing each other. This paper has investigated ecological balance on

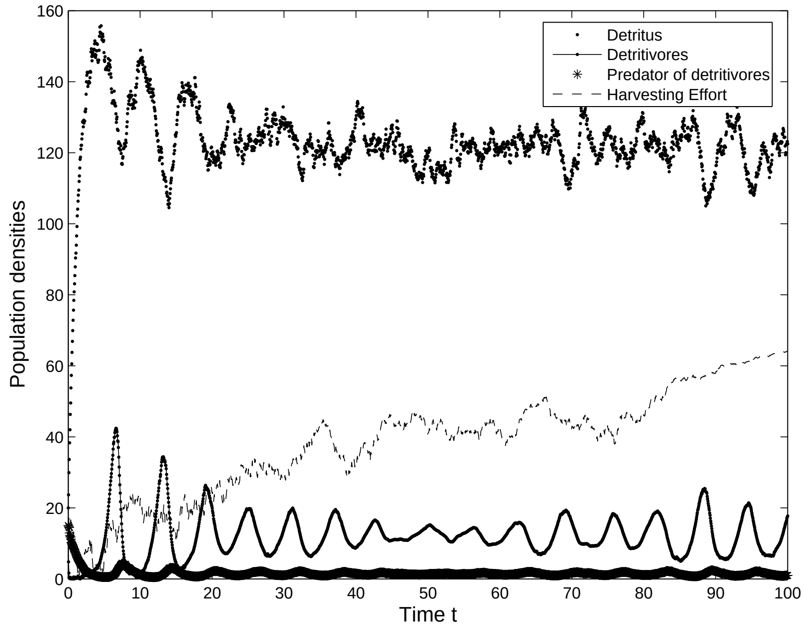

Figure 5: Stable oscillations of stochastic model system (7.1)–(7.4) with small intensity of the noise

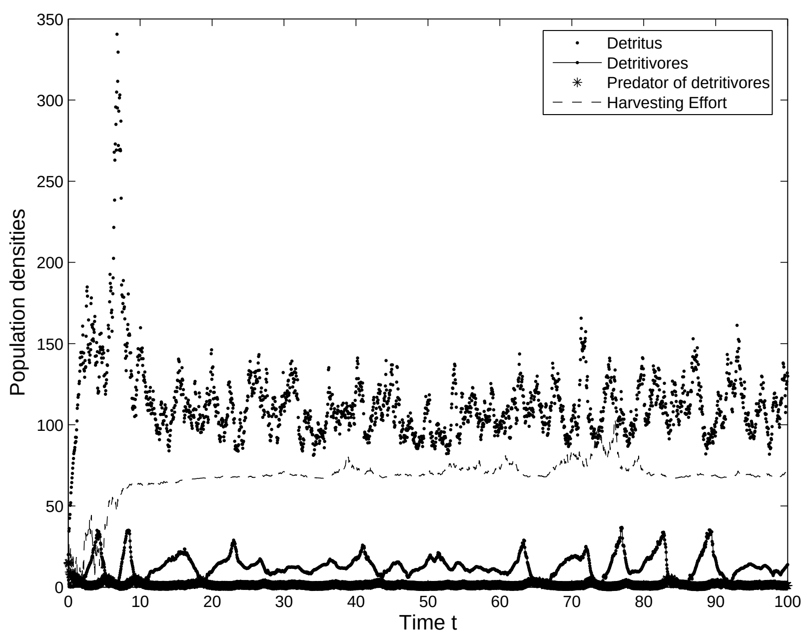

Figure 6: Stable oscillations of stochastic model system (7.1)–(7.4) with high intensity of noise

sensitive parameters which mitigate and adapt to changes. It also professes ideas on how to preserve and protect ecosystems and their services in the real world situation.

In our paper we have observed the local and global behaviours of the control parameters in detritus-based aquatic ecosystem in Sunderban Delta in India. In the mangrove ecosystem we have experienced that the top-down control function for a detritus-based mathematical model is relatively less likely to control the bacterial food supply. Also the interactive effects of nutrient enrichment and changes in species on the functioning of coastal salt marsh ecosystems have a great role to shape the dynamical behaviour of our system. This analysis will contribute to the resolution of a pervasive problem in environmental science including the resolution of physical space with biotic processes which has a major role in complex nonlinear systems.

Mathematical modelling is an important integration tool that is used here to test the understanding at many levels, such as the magnitude changes in the growth rate of individuals due to the increased food supply in a detritus-based ecosystem population. Moreover, in the detritus-based mangrove ecosystem the nutrient enrichments affects the rates of marsh decomposition and the salt marsh landscape. Marine estuaries with large areas of salt marsh are a common feature along the entire Sunderban mangrove area which is the main topic in our paper. Mangroves are rich in microbes especially during the process of leaf decomposition. Detritus processing with the breaking down of organic matter into smaller particles is an essential operation in aquatic systems because it provides resources to filter feeders and accelerates nutrient release by microorganisms. Detrital food web dynamics are influenced by both consumption (top-down) and production (bottom-up) effects. We analyzed the effects of predators and detritivores on the abundance of microorganisms in Sunderban ecosystem delta zone in India. Our study shows that the role of processing detritivores is so complex and can enhance both bottom-up and top-down effects. Specifically, omnivory can complicate simple top-down and bottom-up predictions. Although they accelerate decomposition by microorganisms and thereby can increase resource availability. The processing of detritivores can also be considered important consumers in detrital food webs.

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