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CORRIGENDUM TO "EXISTENCE OF PSEUDO-ALMOST PERIODIC SOLUTIONS OF FUNCTIONAL DIFFERENTIAL EQUATIONS WITH INFINITE DELAY" (APPL. ANAL. 88(12):1713–1726, 2009)

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Abstract. The aim of this note is to show that the main results of the paper "Existence of pseudo-almost periodic solutions of functional differential equations with infinite delay", Applicable Analysis, Vol. 88, No. 12, December 2009, pp. 1713–1726, by Liuwei Zhang and Yuantong Xu, is unfortunately false. The approach proposed by the authors is not correct and the work contains many mistakes.

Keywords: Partial functional differential equation, pseudo almost periodic solution.

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In the cited incorrect paper the authors propose to study the existence of a pseudo almost periodic solution for the following partial functional differential equation

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}u(t) = Au(t) + L(u_t) + f(t), \text{ for } t \geq 0, \\ \quad u_0 = \varphi \in \mathcal{B}, \end{cases} \tag{1}$$

where A is a linear operator on a Banach space X not necessarily densely defined and satisfies the

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Hille-Yosida condition, namely there exist $M \geq 0$ and $\omega \in \mathbb{R}$ such that $(\omega, +\infty) \subset \rho(A)$ and

$$\left|(\lambda - A)^{-n}\right| \leq \frac{M}{(\lambda-\omega)^n} \text{ for } n \in \mathbb{N} \text{ and } \lambda > \omega,$$

where $\rho(A)$ is the resolvent set of A and $\mathcal{B}$ is the phase space which is a complete space consisting of functions from $(-\infty, 0]$ to X. L is a bounded linear operator from $\mathcal{B}$ to X and f is a pseudo almost periodic function from $\mathbb{R}$ to X, the history function $u_t \in C$ is defined by

$$u_t(\theta) = u(t+\theta), \text{ for } \theta \leq 0.$$

The authors state in Theorem 3.4 that the existence of a bounded solution on $\mathbb{R}$ implies the existence of a pseudo almost periodic solution of equation (1). This is false, of course, and we will explain why. The authors proposed to use the reduction principal that has been developed recently in [1]. For that reason, firstly the authors proposed to study the existence of a pseudo almost periodic solution for the following ordinary differential equation

$$\frac{d}{dt}x(t) = Gx(t) + P(t), \ t \in \mathbb{R} \tag{2}$$

where G is a constant $n \times n$ matrix and $P : \mathbb{R} \to \mathbb{R}^n$ is pseudo almost periodic. The main result of this work is Lemma 3.3 which is false. It says that the existence of a bounded solution of equation (2) on $\mathbb{R}$ implies the existence of a pseudo almost periodic solution. In fact, the authors used the following result which is incorrect: In line 2 of the proof of Lemma 3.3, they claimed that the existence of a bounded solution on $\mathbb{R}$ of equation (2) implies that G has no eigenvalues with null real part, and there exists a bounded solution of equation (2) on $\mathbb{R}$. This, of course, is also false, and we can construct easily many examples where the matrix G has eigenvalues with null real parts and equation (2) has a bounded solution on $\mathbb{R}$. In conclusion, the main results of this work is incorrect. The reduction principle does not make sense in this work.

References

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